Qualifying Exam Syllabus

Alexander Shashkov

Monday, April 6, 2026 at 9AM in Evans 730

Committee: Sug Woo Shin (Advisor), Richard Borcherds, Yunqing Tang, Paul Vojta (Chair)

1 Major Topic: Algebraic Geometry (Algebra)

Vakil, *The Rising Sea: Foundations of Algebraic Geometry*, chapters 1–23, excluding starred sections.

1. Schemes: presheaves and sheaves, Spec and Proj, irreducible, reduced, integral, (locally) Noetherian, normal, factorial, regularity, smoothness, (co)-dimension, Algebraic Hartogs Lemma, Bertini's theorem.
2. Morphisms: open/closed/locally closed embeddings, (locally) finite type, finite, separated, proper, affine, projective, quasicompact, quasiseparated, normalization, fibers.
3. Functors: representability, functor of points, fiber products, morphisms to projective space, pushforward, pullback.
4. Commutative algebra miscellanea: Lying over, going up, going down, Nakayama, Krull's theorems, Artin-Rees.
5. Sheaves of modules: (quasi)coherent sheaves, line and vector bundles, (very) ampleness, morphisms to projective space, relative Spec and Proj.
6. Divisors: Weil divisors, Picard group, effective Cartier divisors.
7. Differentials: sheaf of differentials, Euler exact sequence, smoothness.
8. Cohomology: Cech cohomology of quasicoherent sheaves, sheaf cohomology, Hilbert polynomial, statement of Serre duality, derived functors, higher direct images.
9. Curves: Curve-to-Projective Extension Theorem, genus, Riemann-Roch, Riemann-Hurwitz, elliptic and hyperelliptic curves, canonical embedding.

2 Major Topic: Number Theory (Algebra)

Neukirch, *Algebraic Number Theory*, I.1-10, II (except II.6); and Milne, *Class Field Theory*, chapters I, V.

1. Number Fields: Integrality, norm, trace, Dedekind Domains, Minkowski theory, class groups and the class number, Dirichlet's Unit Theorem, ramification theory, discriminant and different, Decomposition and Inertia Group, cyclotomic fields, quadratic number fields.
2. Local Fields: p-adic numbers, local fields, completions, valuations and absolute values, extensions of valuations, ramification (un/tamely/wildly ramified), Hensel's lemma and Henselian fields, Krasner's lemma, Ostrowski's theorem, higher ramification groups (lower numbering).

3. Class Field Theory: Adéles and idéles, product formula, statements of local and global CFT, statement of Artin Reciprocity, Chebotarev density theorem.

3 Minor Topic: Lie Theory (Geometry/Algebra)

Introduction to Lie Groups and Lie Algebras, Kirillov, Chapter 2-7

1. Lie Groups: Basic differential geometry, smooth manifolds, vector fields. Lie groups, relation between lie groups and lie algebras.
2. Lie Algebras: Solvable, nilpotent, semisimple Lie algebras, Lie and Engel's theorem, Killing form, Cartan's Criterion, universal enveloping algebra, statement of PBW Theorem, Casimir element, representations of sl_2, Weyl character/dimension formula.
3. Root Systems: Roots and root spaces, Weyl groups, Cartan matrices, Dynkin diagrams, statement of classification of semisimple Lie algebras over C.