Discussion #2 1/26/26 – Spring 2026 MATH 54
Linear Algebra and Differential Equations

Questions

Problems

1. Answer the following *True* or *False*. Justify your answer.

   (a) The points in the plane corresponding to $\begin{bmatrix} -2 \\ 5 \end{bmatrix}$ and $\begin{bmatrix} -5 \\ 2 \end{bmatrix}$ lie on a line through the origin.

   Solution: False: We require all points on a line through the origin to be multiples of each other.

   (b) An example of a linear combination of vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ is the vector $\frac{1}{2}\mathbf{v}_1$.

   Solution: True: We have
   $$\frac{1}{2}\mathbf{v}_1 = \frac{1}{2}\mathbf{v}_1 + 0 \cdot \mathbf{v}_2.$$

   (c) The solution set of the linear system whose augmented matrix is $[\mathbf{a}_1 \quad \mathbf{a}_2 \quad \mathbf{a}_3 \quad \mathbf{b}]$ is the same as the solution set of the equation
   $$x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + x_3\mathbf{a}_3 = \mathbf{b}.$$

   Solution: True: This is how the matrix-vector multiplication works.

   (d) The set $\text{Span}\{\mathbf{u}, \mathbf{v}\}$ is always visualized as a plane through the origin.

   Solution: False: Consider
   $$\text{Span}\{\mathbf{e}_1, -\mathbf{e}_1\} \quad \text{or} \quad \text{Span}\{\mathbf{e}_1, \mathbf{0}\}.$$

2. Write down a system of 2 equations and 3 unknowns that satisfies each of the following:

   (a) No solution.

   Solution: We can have
   $$\begin{cases} x_1 + x_2 + x_3 = 5 \\ x_1 + x_2 + x_3 = 0 \end{cases}$$
   along with many other possibilities.

   (b) Infinitely many solutions.

   Solution: We can have
   $$\begin{cases} x_1 + x_2 + x_3 = 1 \\ \qquad\; x_2 + x_3 = 0 \end{cases}$$
   along with many other possibilities.

3. Let

$$A = \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 3 \\ 7 \\ -3 \end{bmatrix}.$$

Is $\mathbf{b}$ in the span of A's column vectors?

Solution: No,

$$\left[\begin{array}{ccc|c} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & 7 \\ -2 & 8 & -4 & -3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & 7 \\ 0 & 0 & 0 & 3 \end{array}\right]$$

the augmented matrix $[A \mid \mathbf{b}]$ is not consistent.

Hence, $\mathbf{b}$ is not in the span of A's column vectors.

4. Let

$$\mathbf{u} = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \quad \text{and} \quad \mathbf{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}.$$

Show that $\text{Span}\,\{\mathbf{u}, \mathbf{v}\} = \mathbf{R}^2$. To do this, show that

$$\begin{bmatrix} h \\ k \end{bmatrix} \in \text{Span}\,\{\mathbf{u}, \mathbf{v}\}$$

for all $h, k \in \mathbf{R}$.

Solution: The system

$$\left[\begin{array}{cc|c} 2 & 2 & h \\ -1 & 1 & k \end{array}\right]$$

is consistent for all $h, k \in \mathbf{R}$. Therefore,

$$\text{Span}\,\{\mathbf{u}, \mathbf{v}\} = \mathbf{R}^2$$

5. Let

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}.$$

(a) Compute $A\mathbf{x}$ if

$$\mathbf{x} = \begin{bmatrix} -3 \\ 6 \end{bmatrix}.$$

Solution: We have

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ 6 \end{bmatrix} = \begin{bmatrix} (1\cdot(-3)+2\cdot 6) \\ (3\cdot(-3)+4\cdot 6) \\ (5\cdot(-3)+6\cdot 6) \end{bmatrix} = \begin{bmatrix} (-3+12) \\ (-9+24) \\ (-15+36) \end{bmatrix} = \begin{bmatrix} 9 \\ 15 \\ 21 \end{bmatrix}$$

and this vector lives in $\mathbf{R}^3$.

(b) Explain why $A\mathbf{y}$ does not exist if

$$\mathbf{y} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}.$$

Solution: The matrix A is of size 3×2 and $\mathbf{y}$ is of size 3×1. Since $2 \neq 3$, the inner dimensions do not line up so $A\mathbf{y}$ is not defined. You can also see this by taking the dot product of any of A's rows and recogonize that we cannot dot a vector of length 2 with a vector of length 3.

(c) If A is any 3×2 matrix, what is

$$A\,\mathbf{0}$$

if $\mathbf{0} \in \mathbf{R}^2$?

Solution: The term

$$A\mathbf{x} = \mathbf{0} \in \mathbf{R}^3$$

because we are taking the sum

$$0\mathbf{v}_1 + 0\mathbf{v}_2 = \mathbf{0}.$$

Here each $\mathbf{v}_k$ is the kth column vector of a A,

$$A = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2. \end{bmatrix}$$

6. If

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix} \quad \text{and} \quad \mathbf{x} = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$

how would you define

$$A^2\mathbf{x}?$$

Solution: We know that $A\mathbf{x} = \mathbf{y}$ is some vector in $\mathbf{R}^3$ so we can let

$$A^2\mathbf{x} = A(A\mathbf{x}) = A\mathbf{y}.$$

Here

$$\mathbf{y} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ 3 \end{bmatrix}$$

so

$$A^2\mathbf{x} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 6 \\ 3 \end{bmatrix} = \begin{bmatrix} 11 \\ 18 \\ 9 \end{bmatrix}.$$

7. If the sum of three vectors in $\mathbf{R}^3$ is zero, must they lie in the same plane? Explain.