

Discussion #34 4/27/26 – Spring 2026 MATH 54

Linear Algebra and Differential Equations

Problems

1. (a) Compute the Wronskian of the vector-valued functions

$$\mathbf{x}_1 = \begin{bmatrix} t \\ 1 \end{bmatrix} \quad \text{and} \quad \mathbf{x}_2 = \begin{bmatrix} t^2 \\ 2t \end{bmatrix}.$$

- (b) Are \mathbf{x}_1 and \mathbf{x}_2 linearly independent on $(0, 1)$? On \mathbf{R} ?
- (c) What conclusion can you draw about the coefficients in a system of homogeneous differential equations satisfied by \mathbf{x}_1 and \mathbf{x}_2 ?
- (d) Find a system of homogeneous differential equations satisfied by \mathbf{x}_1 and \mathbf{x}_2 and verify your conclusion in part (c).
2. Part of a mixing device consists of two tanks connected by pipes. Suppose that initially, tank A contains 2 gallons of water with 1 pounds of salt dissolved in it, and tank B contains 2 gallons of pure water.

Suppose that the mixer pipes fluid from A to B at the rate of 1 gallon per minute, and another pipe takes fluid from B to A at the same rate.

- (a) Set up a system of two first order equations and find the amount of salt in each tank as a function of time.
- (b) Will tank B ever have more salt than tank A ? What happens to the two tanks as $t \rightarrow \infty$?
3. Find the real-valued general solution of the system

$$\mathbf{x}'(t) = A\mathbf{x}(t)$$

for

$$A = \begin{bmatrix} -2 & -5 \\ 1 & 2 \end{bmatrix}.$$

4. Solve the following differential equations:
- (a) $y' = 3y$
- (b) $y' = xy$
- (c) $y' + (x - \sigma)y = 0$
5. Evaluate the following partial derivatives:

(a) $\frac{\partial}{\partial x} (x^2y + \cos(y)e^x - 2\sin(x))$

$$(b) \left. \frac{\partial}{\partial t} (\cos(tx)) \right|_{t=0}$$

$$(c) \frac{\partial}{\partial y} (x + 3)$$

$$(d) \frac{\partial}{\partial y} (y^3 - 3 \cos(2y))$$