Discussion #30 4/17/26 – Spring 2026 MATH 54
Linear Algebra and Differential Equations

Problems

1. Find the real-valued, general solution in t of the following:
   (a) $y'' + 2y' = 0$
   (b) $y'' - y' - 12y = 0$
   (c) $y'' - 10y' + 25y = 0$
   (d) $y'' + 9y = 0$

2. Find a fundamental set $\{y_1, y_2\}$ of solutions to the equation $y'' + 4y = 0$ such that

$$y_1(\pi/2) = 1, \quad y_2(\pi/2) = 0,$$
$$y_1'(\pi/2) = 0, \quad y_2'(\pi/2) = 1$$

   Is your set unique?

3. (a) If a, b, and c are positive constants, show that all solutions of $ay'' + by' + cy = 0$ approach zero as $t \to \infty$.
   (b) If $a > 0$, $c > 0$, and $b = 0$, show that the result of part (a) is not true, but that all solutions are bounded as $t \to \infty$.
   (c) Now suppose that $a > 0$, $b > 0$, but that $c = 0$. Show that the result of part (a) is not true, but that all solutions approach a constant that depends on the initial conditions as $t \to \infty$. Determine this constant for the initial conditions $y(0) = y_0$, $y'(0) = y_0'$.

Additional Problem

1. An equation of the form

$$t^2 \frac{d^2y}{dt^2} + \alpha t \frac{dy}{dt} + \beta y = 0, \qquad t > 0 \tag{1}$$

   where α and β are real constants, is called an Euler equation.
   (a) Let $x = \ln t$ and calculate dy/dt and d^2y/dt^2 in terms of dy/dx and d^2y/dx^2.
   (b) Use the results of part (a) to transform Eq. (ii) into

$$\frac{d^2y}{dx^2} + (\alpha - 1)\frac{dy}{dx} + \beta y = 0. \tag{2}$$

   Observe that Eq. (2), has constant coefficients. If $y_1(x)$ and $y_2(x)$ form a fundamental set of solutions of Eq. (2), then $y_1(\ln t)$ and $y_2(\ln t)$ form a fundamental set of solutions of Eq. (1).