Discussion #29 4/13/26 – Spring 2026 MATH 54
Linear Algebra and Differential Equations

Problems

1. (a) How many real eigenvalues, counting multiplicity, will a symmetric 100×100 matrix have?

   (b) Let A be an $n \times n$ symmetric matrix of rank r. Explain why the spectral decomposition of A represents A as the sum of r rank 1 matrices.

2. Consider the following passage from §7.4 of Lay:

   > "The *singular values* of A are the square roots of the eigenvalues of $A^T A$, denoted by $\sigma_1, \ldots, \sigma_n$, and they are arranged in decreasing order. That is, $\sigma_i = \sqrt{\lambda_i}$ for $1 \leq i \leq n$."

   Is an advantage gained by listing the singular values in a specific order?

3. Find the spectral decomposition of

   $$A = \begin{bmatrix} 1 & 1 & 5 \\ 1 & 5 & 1 \\ 5 & 1 & 1 \end{bmatrix}$$

   given

   $$A = PDP^{-1}$$

   where

   $$P = \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \\ 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \end{bmatrix} \quad \text{and} \quad D = \begin{bmatrix} -4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

4. Find the singular values of

   $$A = \begin{bmatrix} 5 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix}$$

5. Find the Singular Value Decomposition of

   $$\begin{bmatrix} 4 & 6 \\ 0 & 4 \end{bmatrix}.$$

6. Suppose A has Singular Value Decomposition $U\Sigma V^T$.

   (a) Find the Singular Value Decomposition for A^{-1}, provided A^{-1} exists.

   (b) Find the Singular Value Decomposition for A^T.