

Discussion #28 4/10/26 – Spring 2026 MATH 54

Linear Algebra and Differential Equations

Problems

- Let A be an $n \times n$ matrix. Answer the following *True* or *False*. Explain your reasoning, or give a counterexample.
 - If A is orthogonally diagonalizable, then A is symmetric.
 - If A is not symmetric, then A has at least one non-real eigenvalue.
 - If A is symmetric with eigenvalue λ repeated 5 times then the eigenspace corresponding to λ has dimension 5.

- Suppose that A is a 2×2 matrix with eigenvalues 0 and 1 and corresponding eigenvectors $(1, 3)$ and $(3, -1)$.

- Is A symmetric?
- Find A and check your answer to part (a).

- Let

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Find a matrix Q that orthogonally diagonalizes A , and determine $Q^{-1}AQ$.

- Let

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}.$$

- What is $\text{rank}(A)$? What is $\dim \text{Nul}(A)$?
 - Why is 0 an eigenvalue of A , and what is the dimension of the eigenspace corresponding to 0?
 - Does A have any other eigenvalues besides 0? Explain.
- Let A be a 4×4 matrix.
 - If the eigenvalues of A are 1, -2 , 3, -3 , can you figure out $\det(A)$?
 - What if the eigenvalues are -1 , 1, 2?
 - What if the eigenvalues are -1 , 0, 1?
 - Prove that if there is an orthogonal matrix that diagonalizes A , then A is symmetric. (See Question 1a.)