Discussion #5 2/2/26 – Spring 2026 MATH 54
Linear Algebra and Differential Equations

Problems

1. Find a basis for the subspace W of $\mathbf{R}^4$ spanned by

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \\ 4 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 3 \\ 2 \\ 1 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 0 \\ 3 \\ 6 \\ 15 \end{bmatrix}$$

2. Show that $\{x^3 + x + 1, 2x^3 + x + 1, x^3 + 3x + 1, x^3 + x + 4\}$ is a linearly dependent set in $\mathbf{P}_3$. (Hint: Find a 3-dimensional subspace that they all lie in.)

3. (a) Show that any two vectors chosen from a linearly independent set are linearly independent.

   (b) Show that a set which contains two linearly dependent vectors must be a linearly dependent set.

   (c) Find three vectors in $\mathbf{R}^3$ which are linearly dependent, and such that any two of them are linearly independent.

4. Let

$$\mathcal{B} = \left\{ \begin{bmatrix} 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 6 \\ 7 \end{bmatrix} \right\}, \quad [\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 8 \\ -5 \end{bmatrix}. \tag{1}$$

   What is the vector $\mathbf{x}$ given by the coordinates $[\mathbf{x}]_{\mathcal{B}}$?

5. Let

$$\mathcal{B} = \left\{ \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ -5 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ -7 \\ 3 \end{bmatrix} \right\}, \quad [\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} -4 \\ 8 \\ -7 \end{bmatrix}. \tag{2}$$

   What is the vector $\mathbf{x}$ given by the coordinates $[\mathbf{x}]_{\mathcal{B}}$?

6. Let

$$\mathcal{B} = \left\{ \begin{bmatrix} 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 6 \\ 7 \end{bmatrix} \right\}, \quad \mathbf{x} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{3}$$

   What are the coordinates of $\mathbf{x}$ with respect to the basis $\mathcal{B}$?

7. Let

$$\mathcal{B} = \left\{ \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ -5 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ -7 \\ 3 \end{bmatrix} \right\}, \mathbf{x} = \begin{bmatrix} 4 \\ -7 \\ 4 \end{bmatrix}. \tag{4}$$

   What are the coordinates of $\mathbf{x}$ with respect to the basis $\mathcal{B}$?