Discussion #12 2/23/26 – Spring 2026 MATH 54
Linear Algebra and Differential Equations

Problems

1. Consider the matrix
$$A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & -1 & -2 \end{bmatrix}.$$
   (a) Find a spanning set for the null space of A.
   (b) Find a spanning set for the column space of A. Can you find a spanning set with only 2 vectors?

2. Let A be an $n \times n$ matrix such that $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.
   (a) What is $\operatorname{Nul} A$?
   (b) What is $\operatorname{Col} A$?

3. Consider the linear transformation $T : \mathbf{R}^3 \to \mathbf{R}^2$ with standard matrix representation
$$A = \begin{bmatrix} 2 & 1 & -1 \\ -4 & -2 & 2 \end{bmatrix}.$$
   (a) Carefully sketch the null space of A. (Of what vector space is it a subspace?)
   (b) Carefully sketch the column space of A. (Of what vector space is it a subspace?)

4. Let
$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$
   (a) Show that the null space of A is the z-axis and the column space of A is the xy-plane.
   (b) Find a 3×3 matrix whose null space is the x-axis and whose column space is the yz-plane.
   (c) Find a matrix whose row space is spanned by $(1, 0, 1)$ and $(0, 1, 0)$ and whose null space is the span of $(1, 0, -1)$.

5. (a) Give an example of a 3×3 matrix whose null space has dimension 1.
   (b) Give an example of a 3×3 matrix whose column space has dimension 1.
   (c) Does there exist a 3×3 matrix whose null space and column space both have dimension 1?

6. (Lay 4.1.40) Let H and K be subspaces of a vector space V. The intersection of H and K, written as $H \cap K$, is the set of $v \in V$ that belong to both H and K.Show that $H \cap K$ is a subspace of V. Give an example in $\mathbb{R}^2$ to show that the union of two subspaces is not, in general, a subspace.

7. Let A be a $n \times n$ matrix such that $A^2 = 0$. Show that $\operatorname{Col} A$ is a subspace of $\operatorname{Nul} A$.