Discussion #10 2/13/26 – Spring 2026 MATH 54
Linear Algebra and Differential Equations

Problems

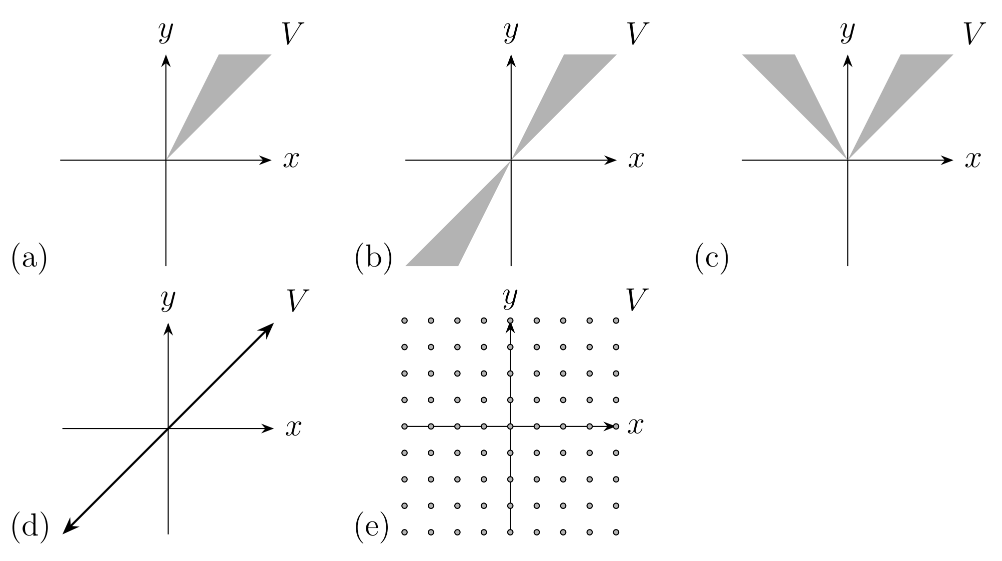
1. Determine if V is closed under addition, scalar multiplication, both, or neither.
   (a) $V = \{\text{odd integers}\}$
   (b) $V = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \text{ are negative} \right\}$
   (c) $V = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ is nonsingular} \right\}$
   (d) $V = \{f \in C[1,2] : f(x) \geq 0 \text{ for all } x \in [1,2]\}$

2. What special properties does the zero vector of a vector space have? Are there other vectors with these same properties? You should try to prove your answer.

3. Let V be the set of all solutions to the differential equation $y'' - 4y' = 0$.
   (a) Is 0 in V?
   (b) If y is a solution to the differential equation, is $2y$ also a solution?
   (c) If y_1 and y_2 are solutions, is $2y_1 + 3y_2$ also a solution?
   (d) Verify that V is a vector space.

4. (a) Let V be a vector space. Is V a subspace of itself?
   (b) If $\mathbf{0}$ is the zero vector in V, is the set $\{\mathbf{0}\}$ a subspace of V?

5. Which regions V are subspaces of $\mathbf{R}^2$?



6. Let V be the vector space of all 2×2 matrices, and let

$$W = \text{Span}\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \right\}$$

   (a) Is $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ in W?

   (b) Is $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ in W?

   (c) Is W a subspace of V?

   (d) What does a typical vector in W look like?

7. Suppose that $\{\mathbf{v}_1, \dots, \mathbf{v}_n\}$ spans the vector space V, and for each i, that $\mathbf{v}_i$ lies in Span $\{\mathbf{w}_1, \dots, \mathbf{w}_m\}$. Show that $\{\mathbf{w}_1, \dots, \mathbf{w}_m\}$ spans V.

   Note: This can be a very useful tool! Suppose you want to know if the space spanned by $\{\mathbf{v}_1, \dots, \mathbf{v}_n\}$ is the same as the space spanned by $\{\mathbf{w}_1, \dots, \mathbf{w}_m\}$. If you can show that each $\mathbf{v}_i$ is in Span $\{\mathbf{w}_1, \dots, \mathbf{w}_m\}$ and each $\mathbf{w}_i$ is in Span $\{\mathbf{v}_1, \dots, \mathbf{v}_n\}$, then the spans must be the same. Make sure you understand this concept before moving on.