

Discussion #2 1/26/26 – Spring 2026 MATH 54

Linear Algebra and Differential Equations

Problems

1. Suppose $T : \mathbf{R}^5 \rightarrow \mathbf{R}^2$ and $T(\mathbf{x}) = A\mathbf{x}$ for some matrix A and for each \mathbf{x} in \mathbf{R}^5 . How many rows and columns does A have?

2. Let

$$A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

Give a geometric description of the transformation $\mathbf{x} \mapsto A\mathbf{x}$.

3. Suppose

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 6x - 2y \\ 3 + y \end{bmatrix}$$

Is T linear?

4. Let

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad \mathbf{v}_1 = \begin{bmatrix} -2 \\ 5 \end{bmatrix}, \quad \text{and} \quad \mathbf{v}_2 = \begin{bmatrix} 7 \\ -3 \end{bmatrix}$$

Define $T : \mathbf{R}^2 \rightarrow \mathbf{R}^2$ by mapping $\mathbf{x} \mapsto x_1\mathbf{v}_1 + x_2\mathbf{v}_2$.

Find a matrix for T .

5. Find matrix that rotates \mathbf{R}^3 in the yz -plane by $\frac{\pi}{3}$ radians counterclockwise.

6. Suppose $T : \mathbf{R}^2 \rightarrow \mathbf{R}^2$ is a linear transformation

$$T\left(\begin{bmatrix} 2 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \text{and} \quad T\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 5 \end{bmatrix}.$$

Find $T\left(\begin{bmatrix} -1 \\ 11 \end{bmatrix}\right)$.

7. Why is the question “Is the linear transformation T onto?” an existence question?

8. Suppose $T : \mathbf{R}^4 \rightarrow \mathbf{R}^6$. Can T be onto?