Discussion #2 1/26/26 – Spring 2026 MATH 54
Linear Algebra and Differential Equations

Questions

Problems

1. Answer the following *True* or *False*. Justify your answer.

   (a) The points in the plane corresponding to $\begin{bmatrix} -2 \\ 5 \end{bmatrix}$ and $\begin{bmatrix} -5 \\ 2 \end{bmatrix}$ lie on a line through the origin.

   (b) An example of a linear combination of vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ is the vector $\frac{1}{2}\mathbf{v}_1$.

   (c) The solution set of the linear system whose augmented matrix is $[\mathbf{a}_1 \quad \mathbf{a}_2 \quad \mathbf{a}_3 \quad \mathbf{b}]$ is the same as the solution set of the equation

   $$x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + x_3\mathbf{a}_3 = \mathbf{b}.$$

   (d) The set $\text{Span}\{\mathbf{u}, \mathbf{v}\}$ is always visualized as a plane through the origin.

2. Write down a system of 2 equations and 3 unknowns that satisfies each of the following:

   (a) No solution.

   (b) Infinitely many solutions.

3. Let

   $$A = \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 3 \\ 7 \\ -3 \end{bmatrix}.$$

   Is $\mathbf{b}$ in the span of A's column vectors?

4. Let

   $$\mathbf{u} = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \quad \text{and} \quad \mathbf{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}.$$

   Show that $\text{Span}\,\{\mathbf{u}, \mathbf{v}\} = \mathbf{R}^2$. To do this, show that

   $$\begin{bmatrix} h \\ k \end{bmatrix} \in \text{Span}\,\{\mathbf{u}, \mathbf{v}\}$$

   for all $h, k \in \mathbf{R}$.

5. Let

   $$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}.$$

(a) Compute $A\mathbf{x}$ if

$$\mathbf{x} = \begin{bmatrix} -3 \\ 6 \end{bmatrix}.$$

(b) Explain why $A\mathbf{y}$ does not exist if

$$\mathbf{y} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}.$$

(c) If A is any 3×2 matrix, what is

$$A\,\mathbf{0}$$

if $\mathbf{0} \in \mathbf{R}^2$?

6. If

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix} \quad \text{and} \quad \mathbf{x} = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$

how would you define

$$A^2\mathbf{x}?$$

7. If the sum of three vectors in $\mathbf{R}^3$ is zero, must they lie in the same plane? Explain.